RESILIENT MODULUS: WHAT, WHY, AND HOW?

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Resilient modulus (M_r) continues to be a key design parameter for pavement systems. Accurate knowledge of the resilient modulus of pavement layer materials allows determination of how the pavement system will respond to traffic loadings. While resilient modulus has been used by many for years, much of its use has been without accurately understanding many key items. Among these items are 1) what is resilient modulus?, 2) how is it determined?, 3) what are typical values?, 4) what influences resilient modulus?, and 5) how is it used in the design of pavement systems? This document will attempt to provide answers to these key questions.

WHAT IS RESILIENT MODULUS?

Most pavement engineering or engineering mechanics textbooks will have a similar definition for resilient modulus. Generally, it is described as the ratio of applied deviator stress to recoverable or "resilient" strain. This definition is correct, but what exactly does it mean in easy to understand terms? To gain a better, more practical understanding of stress and strain consider the following.

Stress

If a given load is applied to a material a contact stress will occur. This stress is equal to the load divided by the loading object's contact area. Stress essentially provides a method of normalizing load and area for testing and design purposes. For example, a 12 x 12 x 12 in. block weighing 200 lbs resting on a soil yields an average contact stress of 200 lbs / (12 x 12 in.) = 1.4 lbs/in^2 or 1.4 psi. As long as the block remains in full contact with the soil, stress will be equal, regardless of soil type.

When a wheel load is applied to a pavement, locations under the load experience different levels of stress based on their depth from the surface and distance from the applied loading. Deviator stress is the axial or vertical stress at a point in the pavement system due to the applied load.

Deformation and Strain

While the stress remains constant, the observed magnitude of soil deformation as a result of loading will likely vary. This deformation may be significant (e.g., block resting on soft soil) or slight (e.g., block resting on stiff soil). In both cases, load has remained constant; however, it is the soil properties that influence deformation. A portion of the deformation may be recoverable or "resilient" while the remainder is unrecoverable or "plastic".

Deformation discussion leads to the critical design variable of strain. Strain is often described as the ratio of an object's deformation to its original dimension in the same direction. Strain can be calculated for any desired direction (e.g., vertical, horizontal, longitudinal, etc.) Consider the block discussed previously. If the block is placed on a 6 inch thick layer of soil and "sinks" 0.25 inches, the total vertical strain in the soil would be (0.25 in. / 6 in.) = 0.042 in./in. or 4.2 percent. If upon removing the block, the soil "rebounds" to a thickness of 5.9 inches, the recoverable or resilient strain would be (5.9 in. – 5.75 in.) / 6 in. = 0.025 in./in. or 2.5 percent. Nonrecoverable or "plastic" strain would be equal to (6 in. – 5.9 in.) / 6 in. = 0.017 in./in. or 1.7 percent. Figure 1 illustrates a typical specimen response during a loading and unloading cycle.

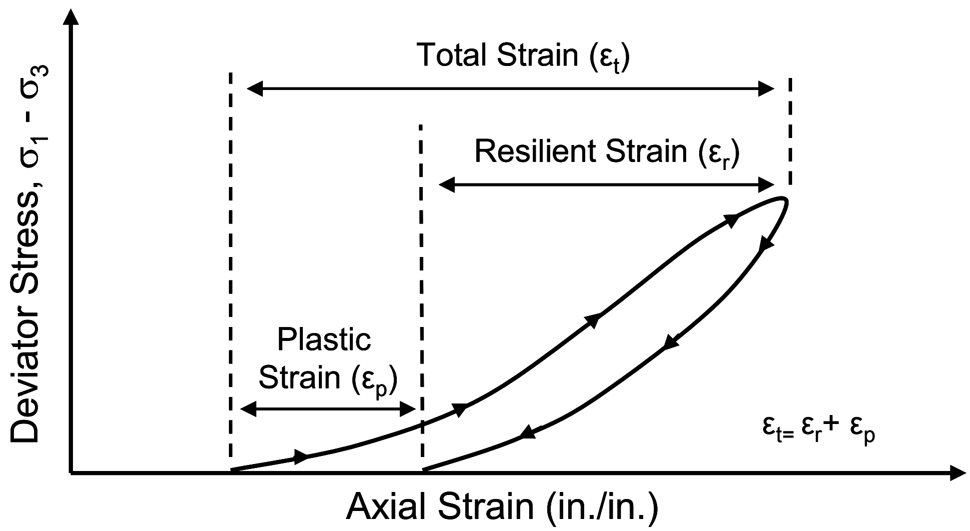

Figure 1 Specimen Response During Axial Loading

Confinement

Pavement materials experience different levels of confinement stress, based upon their position with the pavement structure. Confinement is a result of the surrounding materials and the depth of the material within the pavement structure. This is important because the ability of a granular material to resist loading is, in part, a function of confining stress magnitude.

Stiffness, not Strength

One important item to remember is that resilient modulus is a stiffness measurement, not the strength, of a material. Ultimate shear strength for granular material is typically determined using a triaxial shear procedure. Resilient modulus can be determined at many combinations of applied loading and confinement. Ultimate strength or stress is the point where failure occurs under loading. A good example of the difference between stiffness and strength can be seen with concrete. Up to a given "failure" stress, a concrete can withstand stress with very slight deformation. However, at some stress, the concrete "fails" or "breaks" and the ultimate strength is determined.

Resilient modulus is used to characterize pavement materials under loading conditions that will not result in "failure" of the pavement system. Pavements are designed to withstand various magnitudes of design axle (single, tandem, tridem, and quadem) load applications. By varying layer thicknesses and stiffness, the pavement system can be designed to carry the design axle load applications during its service life.

HOW IS RESILIENT MODULUS MEASURED?

Determination of resilient modulus is generally accomplished through laboratory testing. One commonly used procedure for laboratory testing of soil and aggregate materials is *AASHTO T307, Determining the Resilient Modulus of Soil and Aggregate Materials (1).* During testing, an axial stress is applied for 0.1 second followed by a 0.9 second rest period. Load and rest period together constitutes 1 loading cycle. *Note: The T307 procedure requires aggregate particles greater than 25 percent of the mold diameter (generally 6 inches) be scalped prior to testing. Scalping of "oversize" aggregate may influence the obtained test results.*

One important test procedure aspect is the testing sequences specified for subgrade and subbase/base materials. Different testing sequences, with varying applied and confinement stress, are specified for subgrade and subbase/base materials because the varying stress states experienced under field wheel loading. An illustration of the resilient modulus stress states is provided in Figure 2. Granular material is generally referred to as "stress hardening" material, which means under increased applied stress the material exhibits less deformation and therefore a greater stiffness or resilient modulus. Fine-grained or subgrade soils are referred to as "stress softening", meaning that with increased stress, deformation increases and stiffness or modulus decreases.

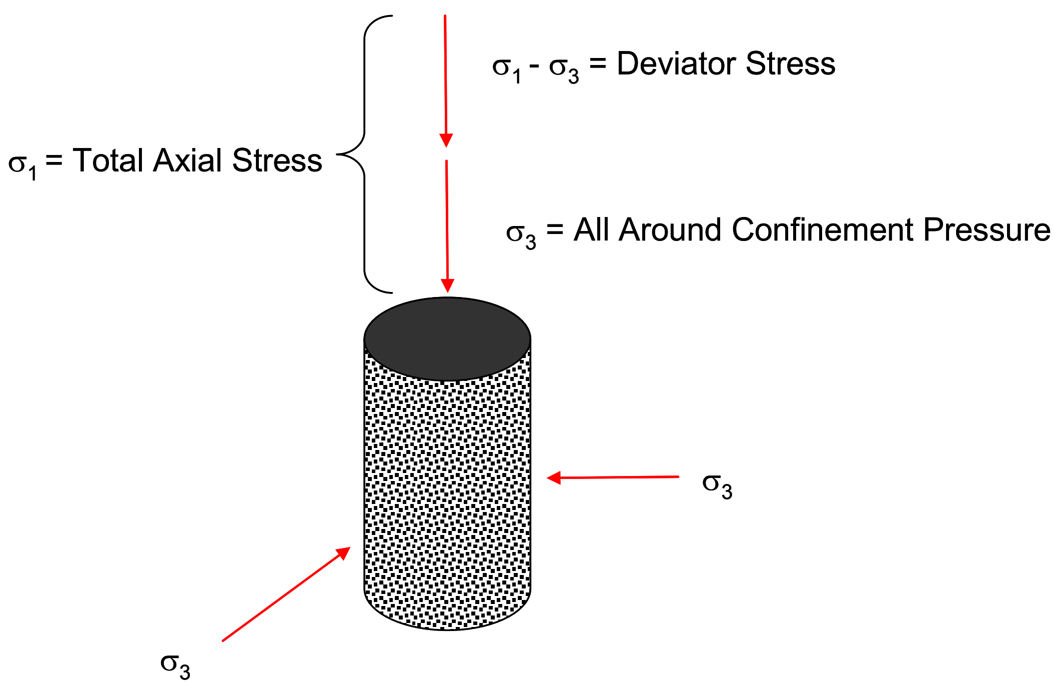

Figure 2 Resilient Modulus Stress States

Prior to actual resilient modulus testing sequence, prepared specimens are conditioned as shown for Sequence "0" in Table 1. Per T307 this conditioning step is for *"elimination of the effects of the interval between compaction and loading and the elimination of initial loading versus* *reloading."* Additionally, this loading serves to minimize the impact of improper contact between the specimen ends and the sample cap and base plate. After conditioning, specified testing sequences from T307 for subgrade and subbase/base materials are shown in Table 1. Subgrade soils are tested at three decreasing levels of confinement (6, 4, and 2 psi) at 5 increasing levels of axial stress (2, 4, 6, 8, and 10 psi) within each confinement stress level. Granular materials are tested at five levels of confinement (3, 5, 10, 15, and 20 psi) with varying levels of axial stress for each confinement level as shown in Table 1. Bulk stress is calculated for each test sequence and represents total specimen stress state. Resilient modulus is then calculated at each of the 15 test sequences. Therefore, a question that must be answered is what is the anticipated stress state or bulk stress of the material in the field? Further, what resilient modulus is applicable for that stress state?

Table 1 AASHTO T307 Test Sequence for Subgrade and Subbase/Base Materials (*1*)

Test Sequence	***SUBGRADE***			***SUBBASE/BASE***		
	Confining Pressure (psi)	***Deviator (Axial) Stress (psi)***	***Bulk Stress (psi)***	***Confining Pressure (psi)***	***Deviator (Axial) Stress (psi)***	***Bulk Stress (psi)***
0	6	4	22.0	15	15	60.0
1	6	2	20.0	3	3	12.0
2	6	4	22.0	3	6	15.0
3	6	6	24.0	3	9	18.0
4	6	8	26.0	5	5	20.0
5	6	10	28.0	5	10	25.0
6	4	2	14.0	5	15	30.0
7	4	4	16.0	10	10	40.0
8	4	6	18.0	10	20	50.0
9	4	8	20.0	10	30	60.0
10	4	10	22.0	15	10	55.0
11	2	2	8.0	15	15	60.0
12	2	4	10.0	15	30	75.0
13	2	6	12.0	20	15	75.0
14	2	8	14.0	20	20	80.0
15	2	10	16.0	20	40	100.0

To determine the resilient modulus at given stress state or bulk stress, it is critical to determine the relationship between resilient modulus and stress state. The "k-θ model" is widely used for granular materials resilient modulus calculation and is shown in Equation 1.

$$M_r = k_1 \theta^{k_2} \qquad \textit{Equation 1}$$

where,

M_r = resilient modulus,

k_1 and k_2 = regression constants,

θ = bulk stress

Regression coefficients, k_1 and k_2, represent the y-intercept and slope, respectively, of a regression line on a log-log plot of resilient modulus versus bulk stress. An example of such a plot for a typical crushed granite base material is shown in Figure 3. With the regression relationship determined, resilient modulus can be selected for appropriate stress state or bulk stress determined from field confinement and applied stress. For example, from Figure 3, for a calculated bulk stress of 30 psi, the resilient modulus would be 28,183 psi.

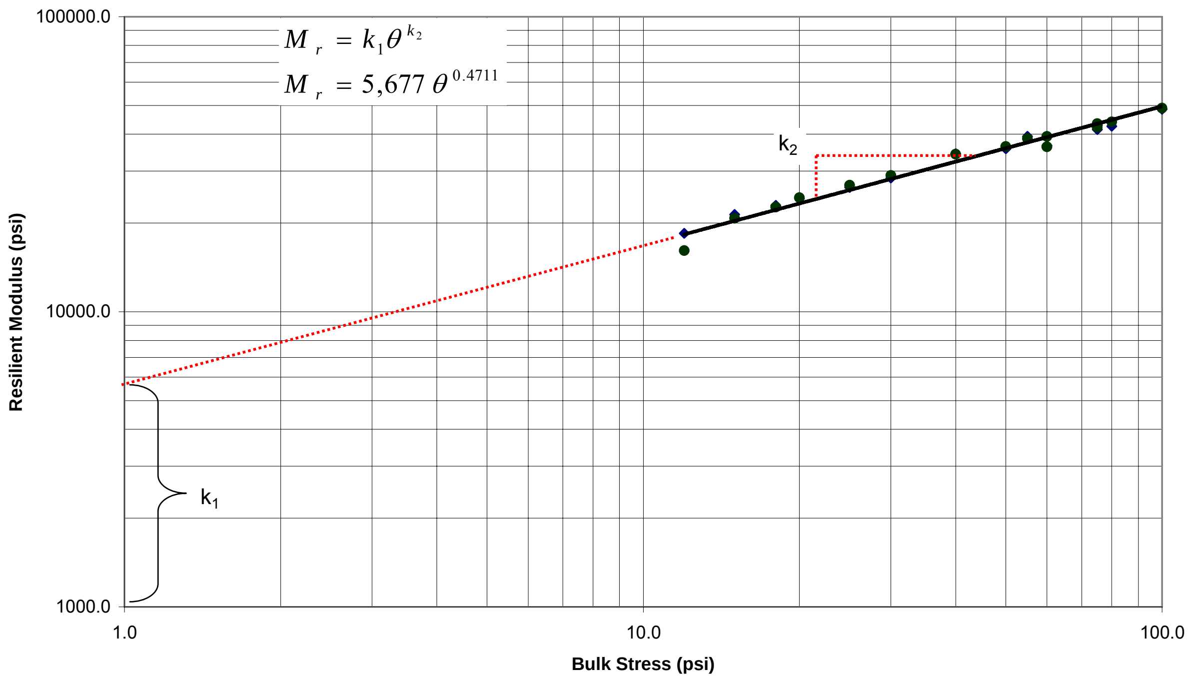

Figure 3 Resilient Modulus versus Bulk Stress

Other resilient modulus models have developed since the introduction of the "k-θ model". The new mechanistic-empirical design guide (M-EPDG) is utilizing the resilient modulus constitutive equation provided in Equation 2 (*2*). This model is commonly referred to as the "k_1-k_3 or universal model" with its main advantage being the consideration of the stress state (i.e., change of normal and shear stress) of the material during testing.

$$M_r = k_1 P_a \left(\frac{\theta}{P_a}\right)^{k_2} \left[\left(\frac{\tau_{oct}}{P_a}\right) + 1\right]^{k_3} \qquad \textit{Equation 2}$$

where,

k_1, k_2, and k_3 = material specific regression coefficients,

θ = bulk stress,

P_a = atmospheric pressure (i.e., 14.7 psi)

τ_{oct} = octahedral shear stress = $\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$

It should be noted that the octahedral shear stress becomes (σ_1 - σ_3) for axisymmetrical stress conditions (i.e., $\sigma_2 = \sigma_3$).

As mentioned, the "k" coefficients for the universal model are determined through regression analysis. The k_1 is proportional to the elastic modulus of the material and will be positive; k_2 is the exponent of the bulk stress term and will be positive since an increasing bulk stress results in a higher modulus; and k_3 is the exponent for the shear stress term and will typically be slightly negative since an increasing shear stress will likely weaken the material resulting in a lower modulus.

WHAT FACTORS INFLUENCE RESILIENT MODULUS?

Compaction

Resilient modulus specimens should be prepared at the target field density to obtain the most realistic estimation of in-place performance. Specimens compacted at a low density will normally have lower resilient moduli than those compacted at a higher density. The magnitude of difference will be a function of many parameters including maximum aggregate size, particle shape, grading (especially fines content), and the applied normal stress.

Specimen initial density can also affect material response to various confining and applied loads. For example, a specimen with low initial density will densify more than one at a high density. This densification will lead to erroneous testing results.

Moisture Content or Degree of Saturation

Moisture plays a key role in material performance. Resilient moduli specimens are typically prepared and tested at their optimum moisture content determined either by Proctor or modified Proctor testing. As with compaction, specimens should be tested in a moisture condition as close as possible field conditions, generally at or very near optimum moisture content. As specimen moisture content increases and degree of saturation approaches 100 percent the resilient modulus will decrease.

Stress State

Appropriate stress state must be calculated so the correct resilient modulus can be determined. Stress state or bulk stress, as previously discussed, is a function of confinement and applied stress. Bulk stress can be calculated as shown in Equation 3.

$$\theta = \sigma_1 + 2\sigma_3 = \sigma_d + 3\sigma_3 \qquad \textit{Equation 3}$$

where,

θ = Bulk stress, psi

σ_1 = Total applied stress, psi

σ_3 = Confinement stress, psi

σ_d = Deviator (Axial) stress or $(\sigma_1 - \sigma_3)$, psi

Using the data provided in Table 1, the bulk stress of 30 psi shown for the subbase/base material for test sequence 6 would be calculated as 15 + (3 x 5) = 30 psi. Within a pavement structure, bulk stress varies as a function of the applied traffic loading, in-situ (i.e., in-place) pavement layer density, and material type. For a given loading, bulk stress decreases as the distance from the pavement surface increases. If an aggregate base layer is placed below a thin HMA layer, the bulk stress in the aggregate layer would be greater compared to it being placed below a thick HMA layer. Consequently, a different resilient modulus should be used for aggregate base in the two cases. An accurate bulk stress must be calculated when selecting the resilient modulus. An example illustrating how to calculate the in-situ stress state for a typical pavement structure and traffic loading is provided below.

IN-SITU STRESS CALCULATION EXAMPLE

To illustrate the procedure for calculation of the in-situ stress state, and therefore bulk stress, consider the example pavement section shown in Figure 4. In Figure 4, pavement layer materials of a given thickness (t) are characterized by an elastic modulus (E), Poisson's ratio (μ), and unit weight (γ). Elastic modulus is a measure of material stiffness and is defined as the slope of the stress-strain curve of a given material within its linear elastic region. Poisson's ratio is the relationship between the lateral (horizontal) and axial (vertical) strain under applied loading and ranges from 0 to 0.5. Resilient modulus is an estimate of the elastic modulus based on recoverable strain under repeated loading.

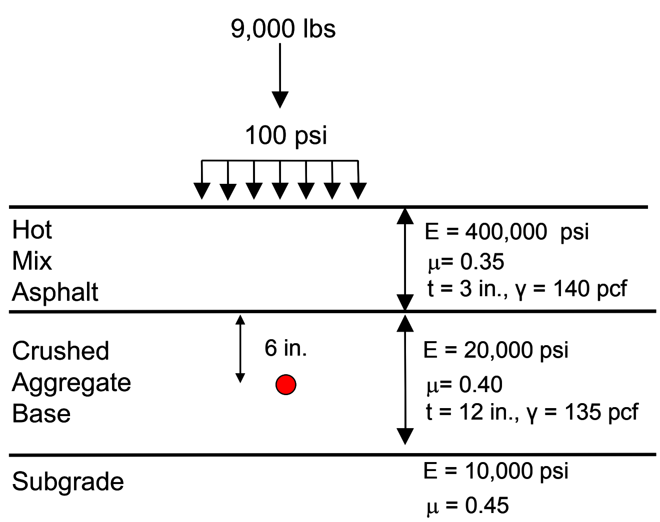

Figure 4Pavement Loading and Cross Section

The steps involved in the calculation of the in-situ stress state are described are as follows (*3*).

1. Calculate the at rest earth pressure coefficient, k_o. For coarse grained aggregate, k_o, the ratio of the horizontal (lateral) to vertical (axial) stress, can be calculated as a function of the angle of internal friction of the aggregate as shown in Equation 4.

$$k_o = 1 - \sin\phi \qquad \textit{Equation 4}$$

$k_o = 1 - \sin 40° = 0.36$ 40° assumed for crushed stone (typical range is 30 to 50°+)

In this example, the vertical stress will be referred as σ_1, with the total vertical stress (σ_{1total}) being the sum of the vertical stresses from the pavement structure (σ_{1pave}) and from the applied load (σ_{1load}). In a similar manner, the horizontal stress will be referred to as σ_3, with the total lateral stress (σ_{3total}) being the sum of the lateral stresses from the pavement structure (σ_{3pave}) and from the applied load (σ_{3load})

2. Determine the at-rest horizontal stress due to the pavement structure at the point of interest (e.g., 6 inches into the crushed aggregate base layer in this case). This at-rest horizontal stress is a result of the unit weight of the pavement layers (γ_p) above the point of interest. It is calculated as a function of the various layer thickness and unit weights, as shown in Equation 5. In-situ horizontal stress (σ_{3pave}) is then calculated as shown in Equation 6.

$$\gamma_p = \frac{\gamma_{HMA}(t_{HMA}) + \gamma_{CAB}(t_{CAB})}{t_{HMA} + t_{CAB}} \qquad \textit{Equation 5}$$

$$\gamma_p = \frac{140\,pcf(0.25\,ft) + 135\,pcf(0.5\,ft)}{(0.25 + 0.5)\,ft} = 136.7\,pcf$$

$$\sigma_{3pave} = k_o\left[\gamma_p(t_{HMA} + t_{CAB})\right] \qquad \textit{Equation 6}$$

$$\sigma_{3pave} = 0.36[136.7\,pcf(0.25\,ft + 0.5\,ft)] = 36.91\,psf = 0.26\,psi$$

3. Determine the horizontal stress as a result of the applied traffic loading (σ_{3load}). This parameter can be calculated using various types of layered elastic analysis (LEA) software. In this example the σ_{3load} is calculated using the Weslea LEA software and determined to be 0.59 psi.

4. Determine the total in-situ horizontal stress from Equation 7.

$$\sigma_{3total} = \sigma_{3pave} + \sigma_{3load} \qquad \textit{Equation 7}$$

$$\sigma_{3total} = 0.26\,psi + 0.59\,psi = 0.85\,psi$$

5. Determine the deviator stresses due to the applied traffic loading (σ_{dload}) as shown in Equation 8.

$$\sigma_{d\,load} = \sigma_{1load} - \sigma_{3\,load} \qquad \textit{Equation 8}$$

The σ_{1load} is calculated using the LEA software mentioned in Step 3. In this example, σ_{1load} is 19.98 psi. The σ_{3load} was previously calculated to be 0.59 psi. Therefore, the σ_{dload} is 19.39 psi as shown below.

$$\sigma_{d\,load} = 19.98\,psi - 0.59\,psi = 19.39\,psi$$

In a similar manner, the in-situ deviator stress due to the pavement structure (σ_{dpave}) can be calculated as provided in Equation 9.

$$\sigma_{d\,pave} = \sigma_{1\,pave} - \sigma_{3\,pave} \qquad \textit{Equation 9}$$

$$\sigma_{d\,pave} = \left(\frac{0.26\,psi}{0.36}\right) - 0.26\,psi = 0.46\,psi$$

6. Calculate the total in-situ deviator stress (σ_{dtotal}) per Equation 10.

$$\sigma_{d\,total} = \sigma_{dload} + \sigma_{dpave} \qquad \textit{Equation 10}$$

$$\sigma_{d} = 19.39\,psi + 0.46\,psi = 19.85\,psi$$

7. Calculate the bulk stress in the pavement structure per Equation 11.

$$\theta = \left(\sigma_{1\,pave} + \sigma_{1load}\right) + 2(\sigma_{3\,pave} + \sigma_{3load}) \qquad \textit{Equation 11}$$

$$\theta = \left(0.72\,psi + 19.98\,psi\right) + 2\left(0.26\,psi + 0.59\,psi\right) = 22.40\,psi$$

8. From the developed relationship for resilient modulus in terms of bulk stress, calculate the appropriate resilient modulus. From Figure 3, an example relationship is provided in Equation 12.

$$M_r = 5{,}677\theta^{0.4711} \qquad \textit{Equation 12}$$

$$M_r = 5{,}677\left(22.40\,psi\right)^{0.4711} = 24{,}560\,psi$$

BULK STRESS VARIATION WITHIN THE PAVEMENT STRUCTURE

The above example illustrates bulk stress and subsequent resilient modulus calculation for one combination of HMA and crushed aggregate base thickness. It has been mentioned earlier that bulk stress varies with respect to position in the pavement structure. Therefore, it is important to understand how bulk stress and modulus change as the respective pavement layer thicknesses vary. Table 2 illustrates the change in bulk stress and modulus for a matrix of HMA and crushed aggregate base thicknesses, based on the preceding example parameters.

Table 2 Bulk Stress and Resilient Modulus for Matrix of HMA and Base Thicknesses

Base (in.)	HMA (in.)									
	2		4		6		8		10	
	Θ (psi)	M_r (psi)	Θ (psi)	M_r (psi)	Θ (psi)	M_r (psi)	Θ (psi)	M_r (psi)	Θ (psi)	M_r (psi)
4	47.92	35,139	26.68	26,670	17.16	21,663	12.30	18,516	9.55	16,434
6	42.08	33,054	23.42	25,079	15.30	20,519	11.18	17,705	8.83	15,841
8	37.57	31,336	20.90	23,771	13.83	19,568	10.28	17,014	8.27	15,363
10	33.79	29,807	18.92	22,679	12.68	18,787	9.58	16,461	7.84	14,976
12	30.46	28,387	17.30	21,745	11.78	18,142	9.02	16,004	7.49	14,661

WHAT VALUE OF RESILIENT MODULUS IS EXPECTED?

It is vital to understand what typical resilient moduli should be expected for various materials. Knowledge of typical values will result in a greater confidence in the pavement design. Many literature sources provide typical ranges for soil and aggregate materials, however, the ranges can be substantially different. Generally for unbound aggregate base materials, the resilient modulus will vary between 15,000 and 60,000 psi. Without conducting resilient modulus testing, an estimate can be obtained using several correlation equations. *Caution: Care should be taken when using correlation equations of any kind. Generally, correlation equations are developed for a limited data set and may not be applicable for a wide range of materials.*

Correlation Methods

Laboratory and field determination of resilient modulus can be time and labor intensive. Therefore, several relationships have been developed in which resilient modulus is calculated as a function of some other material parameter. California Bearing Ratio (CBR), conducted in accordance with AASHTO T193 (*1*), is commonly used to estimate resilient modulus. One common correlation equation using CBR is shown in Equation 13. This equation has been used for fine and coarse aggregate materials; however, it appears to be more applicable for fine grained soils with CBR values less than about 20.

$$M_r\,(psi) = 1500(CBR) \qquad \textit{Equation 13}$$

A more recent correlation, also using CBR, is shown in Equation 142. This equation is being used for resilient modulus prediction in the new Mechanistic-Empirical Pavement Design Guide (M-EPDG), which is currently undergoing final development, calibration, and validation (*2*).

$$M_r(psi) = 2555(CBR)^{0.64} \quad \textit{Equation 14}$$

Equation 15 is another correlation, also in the M-EPDG, that uses the "Hveem Resistance R-value", a measure of stability determined in accordance with AASHTO T190 (*1*).

$$M_r(psi) = 1155 + 555R \quad \textit{Equation 15}$$

In the current AASHTO Guide for the Design of Pavement Structures (*4*), flexible pavements are designed based on a structural number (SN) concept. Based on the associated pavement design parameters (e.g., traffic, reliability, etc.) a required SN is obtained. Pavement layer thicknesses are then determined by using their associated structural layer coefficients (a_i). These coefficients represent the relative strength value for one inch of pavement material. For unbound granular base, a layer coefficient of 0.14 is commonly used in design by agencies. This 0.14 coefficient corresponds to a resilient modulus of approximately 30,000 psi. The above mentioned M-EPDG presents another correlation for modulus as a function of the layer coefficient, as shown in Equation 16. A major disadvantage of Equation 14 is that the layer coefficient has to be determined or estimated prior to estimating the resilient modulus.

$$M_r(psi) = 30{,}000\left(\frac{a_i}{0.14}\right) \quad \textit{Equation 16}$$

HOW IS RESILIENT MODULUS USED IN PAVEMENT DESIGN?

Resilient modulus of pavement materials (HMA, granular base, subgrade, etc.) has been used for many years in structural pavement design. Resilient modulus provides an indication of elastic response of a given material. Elastic materials (e.g., steel) are said to be either linear or nonlinear elastic. Linear elastic materials exhibit a proportional stress-strain relationship and no permanent deformation under load. Nonlinear elastic materials (e.g., concrete) have a nonlinear stress-strain relationship and no observed permanent deformation.

Other materials are referred to as plastic, which means some permanent deformation is observed under loading. Generally, granular bases will exhibit nonlinear elastoplastic behavior in laboratory and field applications. Again, this means the stress-strain relationship is nonlinear and that some amounts of elastic and plastic deformation are present.

Knowing all materials are not purely elastic, the question then becomes why is resilient modulus used for characterization? Layered elastic analysis (LEA) is utilized extensively for pavement system evaluation and is a means of calculating pavement response under loading. Each pavement layer is defined by its resilient modulus and Poisson's ratio. While pavement layer materials are not elastic, LEA is used because it is a relatively simple analysis procedure and, more importantly, pavement loading is generally of low enough magnitude that a linear elastic approximation of pavement material behavior is deemed suitable.

In the new mechanistic-empirical pavement design guide (MEPDG) procedures for flexible pavement design and analysis (see flowchart in Figure 5), LEA is utilized to determine pavement response, based on applied loading, environmental conditions, and material properties (i.e., resilient modulus) at two critical locations: 1) strain at the bottom of the HMA layer and 2) vertical stress at the top of the subgrade. Excessive strain at the bottom of the HMA layer can result in a “fatigue” crack forming and continuing upwards to the pavement surface. Excessive vertical stress at the top of the subgrade can result in permanent or plastic deformation (i.e., rutting) in the subgrade. Over time this rutting will be visible at the pavement surface as a result of support loss.

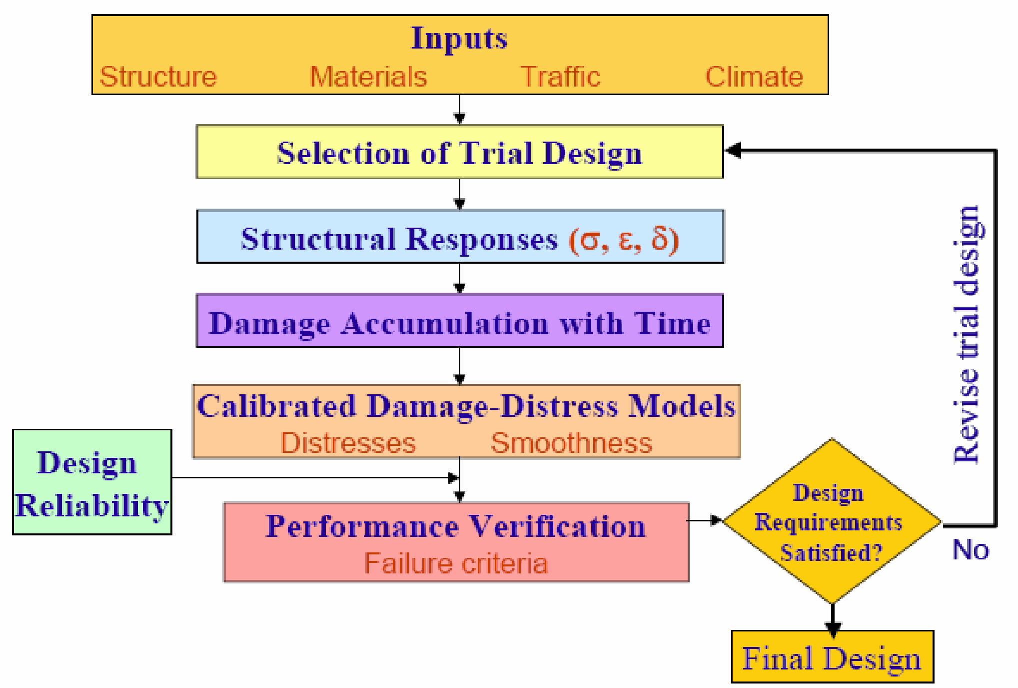

Figure 5 MEPDG Flowchart (5)

FINAL THOUGHTS

Resilient modulus is a key granular material characterization parameter. It is imperative to remember that no “one” resilient modulus exists for a given granular material. Resilient modulus for granular materials is highly stress dependent with the stress state (i.e., bulk stress) being a function of the position of the material in the pavement structure and applied traffic loading. During pavement design, the stress state should be calculated for the given pavement application (structure and loading) and a representative resilient modulus selected.

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4. *AASHTO Guide for Design of Pavement Structures*, American Association of State Highway and Transportation Officials, Washington, D.C., 1993.

5. *2002 Mechanistic Empirical Pavement Design Guide (MEPDG) Software Version 0.910 User's Guide:* Appendix D: User's Guide-Design Guide Software and Design Examples.